BACKPAPER: ALGEBRA I

Date: **Dec 2015**
The Total points is **110** and the maximum you can score is **100** points.

(1) (5+5+5+5=20 points) Prove or disprove
   (a) The groups $(\mathbb{R}, +)$ and $(\mathbb{R}^*, .)$ are isomorphic.
   (b) The groups A_4 and D_{12} are isomorphic.
   (c) Let G be a group and $H \leq G$. Then $H \leq C_G(H)$.
   (d) Let G be a finite group, H be a normal p-sylow subgroup G and $K \trianglelefteq H$ then $K \trianglelefteq G$.

(2) (10 points)Let G be a finite group such that 3 does not divide $|G|$ and for all $a, b \in G$, $(ab)^3 = a^3b^3$. Then G is abelian.

(3) (10 points) Show that the action of D_{2n} on the set $\{1, 2, \ldots, n\}$ given by $\sigma \cdot i = \sigma(i)$ for $\sigma \in D_{2n}$ and $1 \leq i \leq n$ is both transitive and faithful.

(4) (20 points) Show that if $|G| = 385$ then 7-sylow and 11-sylow subgroups are normal and $Z(G)$ contains the 7-sylow subgroup.

(5) (5+15=20 points) Define solvable group. Let p be a prime and $n \geq 1$. Show that any group of order p^n is solvable.

(6) (20 points) Let H be a group. Show that there exist a group G such that $H \trianglelefteq G$ and given any automorphism σ of H there exist a $g \in G$ such that $\sigma(h) = ghg^{-1}$ for all $h \in H$.